Supersymmetry

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Based in part on "A Supersymmetry Primer", `hep-ph/9709356v4` (revised June 2006).

Covered in Lecture 1:

- The Hierarchy Problem, $m_Z \ll m_{\rm Planck}$, is a strong motivation for supersymmetry (SUSY)
- In SUSY, all particles fall into:
  - Chiral supermultiplet $=$ complex scalar boson and fermion partner
  - Gauge supermultiplet $=$ vector boson and gaugino fermion partner
  - Gravitational supermultiplet $=$ graviton and gravitino fermion partner
- The Minimal Supersymmetric Standard Model (MSSM) introduces squarks, sleptons, Higgsinos, gauginos as the superpartners of Standard Model states
- Two-component fermion notation: ψ_α $=$ LH fermion, $\psi^\dagger_{\dot\alpha}$ $=$ RH fermion
- The Wess-Zumino Model Lagrangian describes a single chiral supermultiplet
- The Supersymmetry Algebra
- Superpotentials and interactions

Lecture 2: Masses and Interactions in the Minimal SUSY Standard Model

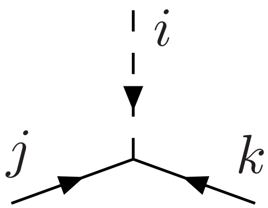
- Supersymmetric gauge interactions
- Soft SUSY breaking
- The MSSM superpotential
- R-parity and its consequences
- Soft SUSY breaking in the MSSM
- What flavor teaches us about SUSY breaking
- Planck-scale Mediated SUSY Breaking
- mSUGRA

Recall: The superpotential $W = M^{ij}\phi^i\phi^j + y^{ijk}\phi_i\phi_j\phi_k$ **determines all non-gauge masses and interactions.**

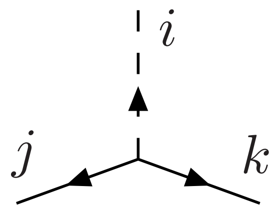
Both scalars and fermions have squared mass matrix $M_{ik}M^{kj}$.

The Feynman rules for our interacting chiral supermultiplets are:

Yukawa interactions:

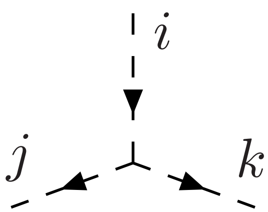


$-iy^{ijk}$

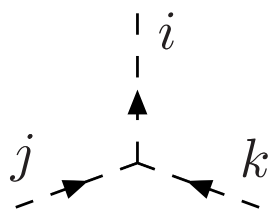


$-iy_{ijk}$

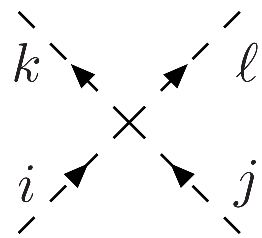
Scalar interactions:



$-iM^{in}y_{njk}$



$-iM_{in}y^{njk}$



$-iy^{ijn}y_{k\ell n}$

Supersymmetric Gauge Theories

A gauge or vector supermultiplet contains physical fields:

- a gauge boson A^a_μ
- a gaugino λ^a_α.

The index a runs over the gauge group generators $[1, 2, \ldots, 8$ for $SU(3)_C$; $1, 2, 3$ for $SU(2)_L$; 1 for $U(1)_Y]$.

Suppose the gauge coupling constant is g and the structure constants of the group are f^{abc}. The Lagrangian for the gauge supermultiplet is:

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu}_a F^a_{\mu\nu} - i\lambda^{\dagger a} \overline{\sigma}^\mu \nabla_\mu \lambda^a + \frac{1}{2} D^a D^a$$

where D^a is a real spin-0 auxiliary field with no kinetic term, and

$$\nabla_\mu \lambda^a \equiv \partial_\mu \lambda^a - g f^{abc} A^b_\mu \lambda^c$$

The auxiliary field D^a is again needed so that the SUSY algebra closes on-shell. Counting fermion and boson degrees of freedom on-shell and off-shell:

	A_μ	λ	D
on-shell ($n_B = n_F = 2$)	2	2	0
off-shell ($n_B = n_F = 4$)	3	4	1

To make a gauge-invariant supersymmetric Lagrangian involving both gauge and chiral supermultiplets, one must turn the ordinary derivatives into covariant ones:

$$\partial_\mu \phi_i \quad \rightarrow \quad \nabla_\mu \phi_i = \partial_\mu \phi_i + igA^a_\mu (T^a \phi)_i$$

$$\partial_\mu \psi_i \quad \rightarrow \quad \nabla_\mu \psi_i = \partial_\mu \psi_i + igA^a_\mu (T^a \psi)_i$$

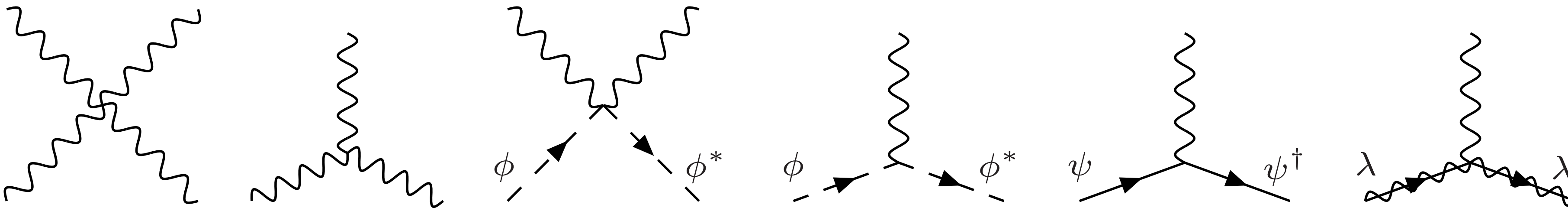
One must also add three new terms to the Lagrangian:

$$\mathcal{L} \quad = \quad \mathcal{L}_{\rm gauge} + \mathcal{L}_{\rm chiral} - \sqrt{2} g (\phi^* T^a \psi)\lambda^a - \sqrt{2} g \lambda^{\dagger a} (\psi^\dagger T^a \phi) + g(\phi^* T^a \phi) D^a .$$

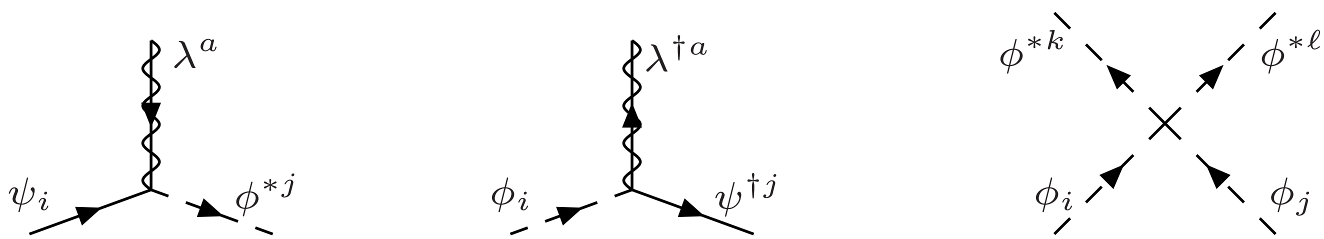
You can check (after some algebra) that this full Lagrangian is now invariant under both SUSY transformations and gauge transformations.

Supersymmetric gauge interactions

The following interactions are dictated by ordinary gauge invariance alone:



SUSY also predicts interactions that have gauge coupling strength, but are not gauge interactions in the usual sense:



$-i\sqrt{2}g_a(T^a)_i{}^j$ $\quad$ $-i\sqrt{2}g_a(T^a)_i{}^j$ $\quad$ $-ig_a^2(T_i^{ak}T_j^{a\ell}+T_i^{a\ell}T_j^{ak})$

These interactions are entirely determined by supersymmetry and the gauge group. Experimental measurements of the magnitudes of these couplings will provide an important test that we really have SUSY.

Soft SUSY-breaking Lagrangians

It has been shown that the quadratic sensitivity to $M_{\rm UV}$ is still absent in SUSY theories with these SUSY-breaking terms added in:

$$
\begin{aligned}
\mathcal{L}_{\rm soft} &= -\tfrac{1}{2}\left(M_a\,\lambda^a\lambda^a + \text{c.c.}\right) - (m^2)^i_j \phi^{*j}\phi_i \\
&\quad -\left(\tfrac{1}{2}b^{ij}\phi_i\phi_j + \tfrac{1}{6}a^{ijk}\phi_i\phi_j\phi_k + \text{c.c.}\right),
\end{aligned}
$$

They consist of:

- gaugino masses M_a,
- scalar (mass)^2 terms $(m^2)^j_i$ and b^{ij},
- (scalar)^3 couplings a^{ijk}

How to make a realistic SUSY Model:

- Choose a gauge symmetry group.

  (In the MSSM, this is already done: $SU(3)_C \times SU(2)_L \times U(1)_Y$.)

- Choose a superpotential W; must be invariant under the gauge symmetry.

  (In the MSSM, this is almost already done: Yukawa couplings are dictated by the observed fermion masses.)

- Choose a soft SUSY-breaking Lagrangian, or else choose a method for spontaneous SUSY breakdown.

  (This is where almost all of the arbitrariness in the MSSM is.)

Let's do this for the MSSM now, and then explore the consequences.

The Superpotential for the Minimal SUSY Standard Model:

$$W_{\rm MSSM} = \tilde{\bar{u}}\,\mathbf{y_u}\tilde{Q}H_u - \tilde{\bar{d}}\,\mathbf{y_d}\tilde{Q}H_d - \tilde{\bar{e}}\,\mathbf{y_e}\tilde{L}H_d + \mu H_u H_d$$

The objects H_u, H_d, $\tilde{Q}$, $\tilde{L}$, $\tilde{\bar{u}}$, $\tilde{\bar{d}}$, $\tilde{\bar{e}}$ appearing here are the scalar fields appearing in the left-handed chiral supermultiplets. Recall that $\bar{u}$, $\bar{d}$, $\bar{e}$ are the conjugates of the right-handed parts of the quark and lepton fields.

The dimensionless Yukawa couplings $\mathbf{y_u}$, $\mathbf{y_d}$ and $\mathbf{y_e}$ are 3×3 matrices in family space. Up to a normalization, and higher-order quantum corrections, they are the same as in the Standard Model. (All gauge and family indices are suppressed.)

We need both H_u and H_d, because terms like $\tilde{\bar{u}}\,\mathbf{y_u}\tilde{Q}H_d^*$ and $\tilde{\bar{d}}\,\mathbf{y_d}\tilde{Q}H_u^*$ are not analytic, and so not allowed in the superpotential.

In the approximation that only the t, b, τ Yukawa couplings are included:

$$\mathbf{y_u} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_t \end{pmatrix}; \qquad \mathbf{y_d} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_b \end{pmatrix}; \qquad \mathbf{y_e} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$

the superpotential becomes

$$\begin{aligned} W_{\rm MSSM} \;\approx\; & y_t(\bar{t}tH_u^0 - \bar{t}bH_u^+) - y_b(\bar{b}tH_d^- - \bar{b}bH_d^0) \\ & -y_\tau(\bar{\tau}\nu_\tau H_d^- - \bar{\tau}\tau H_d^0) + \mu(H_u^+ H_d^- - H_u^0 H_d^0) \end{aligned}$$

Here the $\tilde{}$ are omitted to reduce clutter, and $Q_3 = (t\, b)$; $L_3 = (\nu_\tau\ \tau)$; $H_u = (H_u^+\ H_u^0)$; $H_d = (H_d^0\ H_d^-)$ $\bar{u}_3 = \bar{t}$; $\bar{d}_3 = \bar{b}$; $\bar{e}_3 = \bar{\tau}$.

The minus signs are arranged so that if the neutral Higgs scalars get positive VEVs $\langle H_u^0 \rangle = v_u$ and $\langle H_d^0 \rangle = v_d$, and the Yukawa couplings are defined positive, then the fermion masses are also positive:

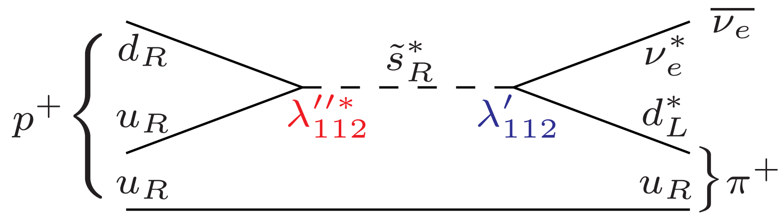
$$m_t = y_t v_u; \qquad m_b = y_b v_d; \qquad m_\tau = y_\tau v_d.$$

Actually, the most general possible superpotential would also include:

$$W_{\Delta L=1} = \frac{1}{2}\lambda_{ijk} L_i L_j \bar{e}_k + \lambda'_{ijk} L_i Q_j \bar{d}_k + \mu'_i L_i H_u$$
$$W_{\Delta B=1} = \frac{1}{2}\lambda''_{ijk} \bar{u}_i \bar{d}_j \bar{d}_k$$

These violate lepton number ($\Delta L = 1$) or baryon number ($\Delta B = 1$).

If both types of couplings were present, and of order 1, then the proton would decay in a tiny fraction of a second through diagrams like this:



Many other proton decay modes, and other experimental limits on B and L violation, give strong constraints on these terms in the superpotential.

One cannot simply require B and L conservation, since they are already known to be violated by non-perturbative electroweak effects. Instead, in the MSSM, one postulates a new discrete symmetry called **Matter Parity**, also known as **R-parity**.

Matter parity is a multiplicatively conserved quantum number defined as:

$$P_M = (-1)^{3(B-L)}$$

for each particle in the theory. All quark and lepton supermultiplets carry $P_M = -1$, and the Higgs and gauge supermultiplets carry $P_M = +1$. This eliminates all of the dangerous $\Delta L = 1$ and $\Delta B = 1$ terms from the superpotential, saving the proton.

R-parity is defined for each particle with spin S by:

$$P_R = (-1)^{3(B-L)+2S}$$

This is **exactly equivalent** to matter parity, because the product of $(-1)^{2S}$ is always $+1$ for any interaction vertex that conserves angular momentum.

However, particle within the same supermultiplet do not carry the same R-parity. You can check that all of the known Standard Model particles and the Higgs scalar bosons carry $P_R = +1$, while all of the squarks and sleptons and higgsinos and gauginos carry $P_R = -1$.

Consequences of R-parity

The particles with odd R-parity ($P_R = -1$) are the “supersymmetric particles” or “sparticles”.

Every interaction vertex in the theory must contain an even number of $P_R = -1$ sparticles. Three extremely important consequences:

- The lightest sparticle with $P_R = -1$, called the “Lightest Supersymmetric Particle” or LSP, must be absolutely stable. If the LSP is electrically neutral, it interacts only weakly with ordinary matter, and so can make an attractive candidate for the non-baryonic dark matter required by cosmology.
- In collider experiments, sparticles can only be produced in even numbers (usually two-at-a-time).
- Each sparticle other than the LSP must eventually decay into a state that contains an odd number of LSPs (usually just one). The LSP escapes the detector, with a missing momentum signature.

The Lightest SUSY Particle as Cold Dark Matter

Recent results in experimental cosmology suggest the existence of cold dark matter with a density:

$$\Omega_{\text{CDM}} h^2 = 0.11 \pm 0.02 \qquad \text{(WMAP 2003)}$$

where h = Hubble constant in units of 100 km/(sec Mpc).

A stable particle which freezes out of thermal equilibrium will have $\Omega h^2 = 0.11$ today if its thermal-averaged annihilation cross-section is, roughly:

$$\langle \sigma v \rangle = 1 \text{ pb}$$

As a crude estimate, a weakly interacting particle that annihilates in collisions with a characteristic mass scale M will have

$$\langle \sigma v \rangle \sim \frac{\alpha^2}{M^2} \sim 1 \text{ pb} \left(\frac{150 \text{ GeV}}{M} \right)^2$$

So, a stable, weakly interacting particle with mass of order 100 GeV is a likely candidate. In particular, a neutralino LSP ($\tilde{N}_1$) may do it, if R-parity is conserved.

Is R-parity inevitable?

No. But, besides saving the proton and giving us a dark matter particle, it has the nice feature that it could arise naturally as a surviving subgroup of a continuous gauge symmetry. If $U(1)_{B-L}$ symmetry is gauged, and then broken at very high energy by a VEV of some field that carries an even integer value of $3(B-L)$, then matter parity will automatically be an exact symmetry of the MSSM.

However, there are alternatives to R-parity, for example **baryon triality**, a Z_3 discrete symmetry:

$$Z_3^B = e^{2\pi i(B-2Y)/3}$$

If Z_3^B is multiplicatively conserved, then the proton is absolutely stable, but the LSP is not.

The Soft SUSY-breaking Lagrangian for the MSSM

$$\begin{aligned}
\mathcal{L}_{\text{soft}}^{\text{MSSM}} \quad = \quad & -\tfrac{1}{2}\left(M_3 \widetilde{g}\widetilde{g} + M_2 \widetilde{W}\widetilde{W} + M_1 \widetilde{B}\widetilde{B}\right) + \text{c.c.} \\
& -\left(\widetilde{\overline{u}}\,\mathbf{a_u}\,\widetilde{Q}H_u - \widetilde{\overline{d}}\,\mathbf{a_d}\,\widetilde{Q}H_d - \widetilde{\overline{e}}\,\mathbf{a_e}\,\widetilde{L}H_d\right) + \text{c.c.} \\
& -\widetilde{Q}^\dagger\,\mathbf{m^2_{\tilde{Q}}}\,\widetilde{Q} - \widetilde{L}^\dagger\,\mathbf{m^2_{\tilde{L}}}\,\widetilde{L} - \widetilde{\overline{u}}\,\mathbf{m^2_{\tilde{\bar{u}}}}\,\widetilde{\overline{u}}^\dagger - \widetilde{\overline{d}}\,\mathbf{m^2_{\tilde{\bar{d}}}}\,\widetilde{\overline{d}}^\dagger - \widetilde{\overline{e}}\,\mathbf{m^2_{\tilde{\bar{e}}}}\,\widetilde{\overline{e}}^\dagger \\
& - m^2_{H_u} H_u^* H_u - m^2_{H_d} H_d^* H_d - (b H_u H_d + \text{c.c.})\,.
\end{aligned}$$

The first line gives masses to the MSSM gauginos (gluino $\widetilde{g}$, winos $\widetilde{W}$, bino $\widetilde{B}$).

The second line consists of (scalar)^3 interactions.

The third line is (mass)^2 terms for the squarks and sleptons.

The last line is Higgs (mass)^2 terms.

If SUSY is to solve the Hierarchy Problem, we expect:

$$M_1,\ M_2,\ M_3,\ \mathbf{a_u},\ \mathbf{a_d},\ \mathbf{a_e}\ \sim\ m_{\text{soft}};$$

$$\mathbf{m^2_{\tilde{Q}}},\ \mathbf{m^2_{\tilde{L}}},\ \mathbf{m^2_{\tilde{\bar{u}}}},\ \mathbf{m^2_{\tilde{\bar{d}}}},\ \mathbf{m^2_{\tilde{\bar{e}}}},\ m^2_{H_u},\ m^2_{H_d},\ b\ \sim\ m^2_{\text{soft}}$$

where $m_{\text{soft}}\ \lesssim\ 1$ TeV.

The soft SUSY-breaking Lagrangian of the MSSM contains 105 new parameters not found in the Standard Model.

Most of what we do not already know about SUSY is expressed by the question: "How is supersymmetry broken?"

Many proposals have been made.

The question can be answered experimentally by discovering the pattern of Higgs and squark and slepton and gaugino masses, because they are the main terms in the SUSY-breaking Lagrangian.

Electroweak symmetry breaking and the Higgs bosons

In SUSY, there are two complex Higgs scalar doublets, (H_u^+, H_u^0) and (H_d^0, H_d^-), rather than one in the Standard Model.

The Higgs VEVs can be parameterized:

$$v_u = \langle H_u^0 \rangle, \qquad v_d = \langle H_d^0 \rangle, \qquad \text{where}$$
$$v_u^2 + v_d^2 = v^2 = 2m_Z^2/(g^2 + g'^2) \approx (175\,\text{GeV})^2$$
$$\tan\beta = v_u/v_d.$$

The quark and lepton masses are related to these VEVs and the superpotential Yukawa couplings by:

$$y_t = \frac{m_t}{v \sin\beta}, \qquad y_b = \frac{m_b}{v \cos\beta}, \qquad y_\tau = \frac{m_\tau}{v \cos\beta}, \qquad \text{etc.}$$

If we want the Yukawa couplings to avoid getting non-perturbatively large up to very high scales, we need:

$$1.5 \lesssim \tan\beta \lesssim 55$$

Define mass-eigenstate Higgs bosons: h^0, H^0, A^0, G^0, H^+, G^+ by:

$$\begin{pmatrix} H_u^0 \\ H_d^0 \end{pmatrix} = \begin{pmatrix} v_u \\ v_d \end{pmatrix} + \frac{1}{\sqrt{2}} \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} h^0 \\ H^0 \end{pmatrix} + \frac{i}{\sqrt{2}} \begin{pmatrix} \sin\beta & \cos\beta \\ -\cos\beta & \sin\beta \end{pmatrix} \begin{pmatrix} G^0 \\ A^0 \end{pmatrix}$$

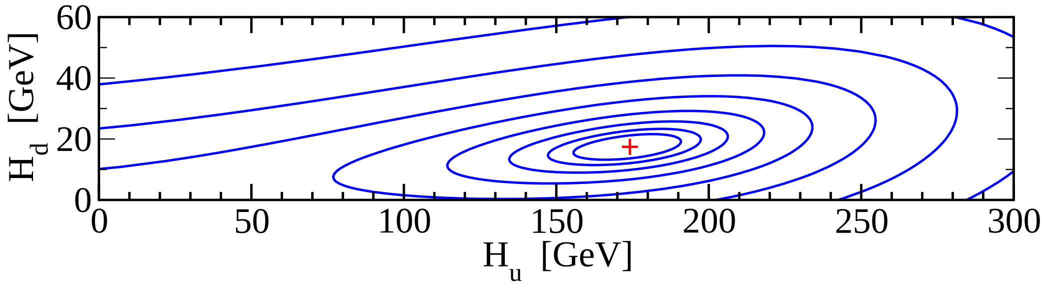
$$\begin{pmatrix} H_u^+ \\ H_d^{-*} \end{pmatrix} = \begin{pmatrix} \sin\beta & \cos\beta \\ -\cos\beta & \sin\beta \end{pmatrix} \begin{pmatrix} G^+ \\ H^+ \end{pmatrix}$$

Now, expand the potential to second order in these fields to obtain the masses:

$$\begin{aligned} m_{A^0}^2 &= 2b/\sin 2\beta \\ m_{h^0,H^0}^2 &= \tfrac{1}{2}\Big(m_{A^0}^2 + m_Z^2 \mp \sqrt{(m_{A^0}^2 + m_Z^2)^2 - 4m_Z^2 m_{A^0}^2 \cos^2 2\beta}\Big), \\ m_{H^\pm}^2 &= m_{A^0}^2 + m_W^2 \end{aligned}$$

The Goldstone bosons have $m_{G^0} = m_{G^\pm} = 0$; they are absorbed by the Z, $W^\pm$ bosons to give them masses, as in the Standard Model.

Typical contour map of the Higgs potential in SUSY:



The Standard Model-like Higgs boson h^0 corresponds to oscillations along the shallow direction with $(H_u^0 - v_u, H_d^0 - v_d) \propto (\cos\alpha, -\sin\alpha)$. At tree-level, it is easy to show from above that the lightest Higgs scalar obeys:

$$m_{h^0} < m_Z.$$

This has been ruled out by LEP2. However, taking into account loop effects, m_{h^0} is considerably larger. Assuming that all superpartners are lighter than 1000 GeV, and that perturbation theory is valid up to $M_{\rm GUT}$, one finds:

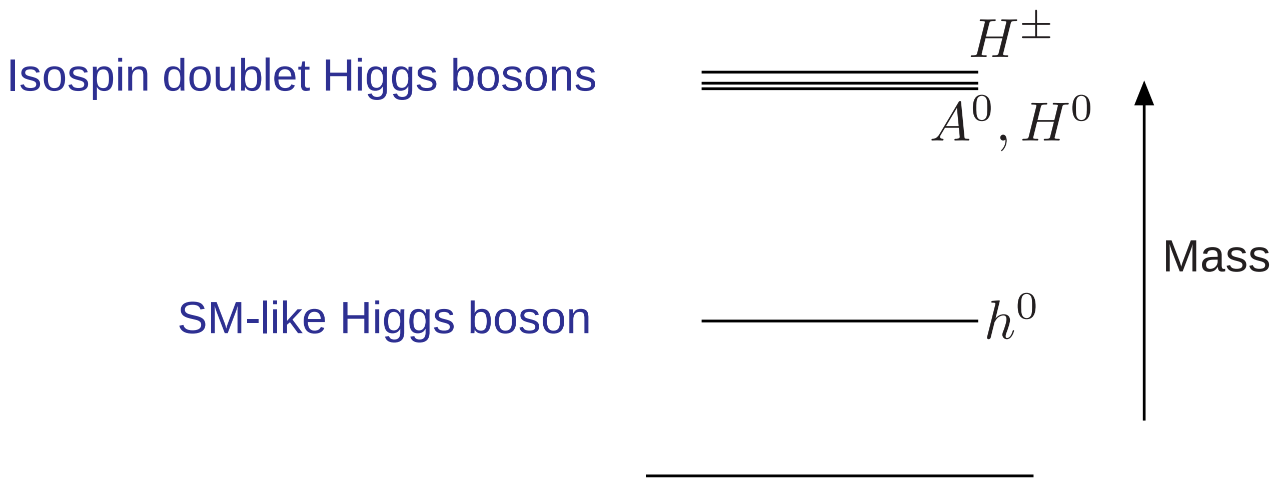
$$m_{h^0} \lesssim 130 \text{ GeV}$$

in the MSSM. By adding more supermultiplets, the bound increases to 150 GeV. By not requiring that the theory stays perturbative, one can get up to 200 GeV.

The decoupling limit for the Higgs bosons

If $m_{A^0} \gg m_Z$, then:

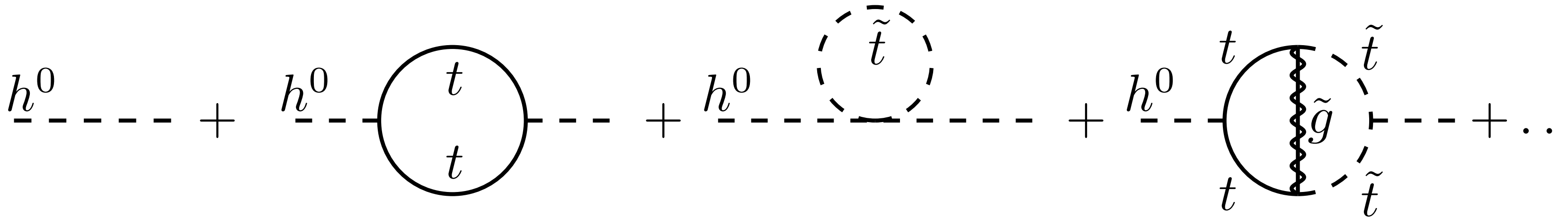
- h^0 has the same couplings as would a Standard Model Higgs boson of the same mass
- $\alpha \approx \beta - \pi/2$
- $A^0, H^0, H^\pm$ form an isospin doublet, and are much heavier than h^0



Many models of SUSY breaking approximate this decoupling limit.

Radiative corrections to the Higgs mass in SUSY:

$$m_{h^0}^2 = m_Z^2 \cos^2(2\beta) + \frac{3}{4\pi^2} y_t^2 m_t^2 \ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right) + \ldots$$



At tree-level: m_Z^2 pure electroweak

At one-loop: $y_t^2 m_t^2$ top Yukawa comes in

At two-loop: $\alpha_S y_t^2 m_t^2$ SUSYQCD comes in

At three-loop: $\alpha_S^2 y_t^2 m_t^2$

Even the three-loop corrections can add 1 GeV or so to m_{h^0}.

This is larger than the experimental uncertainty expected at the LHC.

Neutralinos

The neutral higgsinos ($\tilde{H}_u^0$, $\tilde{H}_d^0$) and the neutral gauginos ($\tilde{B}$, $\tilde{W}^0$) mix with each other after electroweak symmetry breaking to form four **neutralino** fermion states. In the gauge eigenstate basis $\psi_i^0 = (\tilde{B}, \tilde{W}^0, \tilde{H}_d^0, \tilde{H}_u^0)$ for $i = 1, 2, 3, 4$, the neutralino mass terms in the Lagrangian are

$$\mathcal{L}_{\text{neutralino mass}} = -\tfrac{1}{2}(\psi^0)^T \mathbf{M}_{\tilde{N}} \psi^0$$

$$\mathbf{M}_{\tilde{N}} = \begin{pmatrix} M_1 & 0 & -g' v_d/\sqrt{2} & g' v_u/\sqrt{2} \\ 0 & M_2 & g v_d/\sqrt{2} & -g v_u/\sqrt{2} \\ -g' v_d/\sqrt{2} & g v_d/\sqrt{2} & 0 & -\mu \\ g' v_u/\sqrt{2} & -g v_u/\sqrt{2} & -\mu & 0 \end{pmatrix}$$

The diagonal terms are just the gaugino masses in the soft SUSY-breaking Lagrangian. The $-\mu$ entries can be traced back to the superpotential. The off-diagonal terms come from the gaugino-Higgs-Higgsino interactions, and are always less than m_Z.

The physical neutralino mass eigenstates $\tilde{N}_i$ (another popular notation is $\tilde{\chi}_i^0$) are obtained by diagonalizing the mass matrix with a unitary matrix.

$$\tilde{N}_i \quad = \quad \mathbf{N}_{ij}\psi_j^0,$$

where

$$\text{diag}(m_{\tilde{N}_1}, m_{\tilde{N}_2}, m_{\tilde{N}_3}, m_{\tilde{N}_4}) = \mathbf{N}^*\mathbf{M}\mathbf{N}^{-1},$$

with $m_{\tilde{N}_1} < m_{\tilde{N}_2} < m_{\tilde{N}_3} < m_{\tilde{N}_4}$.

In many models of SUSY breaking, one finds:

$$M_1 \approx 0.5 M_2 < |\mu| \qquad \text{and} \qquad m_Z \ll |\mu|$$

where the "0.5" is really $\frac{5}{3}\tan^2\theta_W$. In that case, the lightest neutralino state $\tilde{N}_1$ is mostly bino, with mass nearly equal to M_1.

The lightest neutralino fermion, $\tilde{N}_1$, is a likely candidate for the cold dark matter that seems to be required by cosmology.

Charginos

Similarly, the charged higgsinos H_u^+, H_d^- and the charged winos W^+, W^- mix to form **chargino** fermion mass eigenstates.

$$\mathcal{L}_{\text{chargino mass}} = -\tfrac{1}{2}(\psi^\pm)^T \mathbf{M}_{\widetilde{C}} \psi^\pm + \text{c.c.}$$

where, in 2×2 block form,

$$\mathbf{M}_{\widetilde{C}} \quad = \quad \begin{pmatrix} \mathbf{0} & \mathbf{X}^T \\ \mathbf{X} & \mathbf{0} \end{pmatrix} \qquad \text{with} \qquad \mathbf{X} = \begin{pmatrix} M_2 & gv_u \\ gv_d & \mu \end{pmatrix}$$

The mass eigenstates $\widetilde{C}^\pm_{1,2}$ (many other sources use $\tilde{\chi}^\pm_{1,2}$) are related to the gauge eigenstates by two unitary 2×2 matrices $\mathbf{U}$ and $\mathbf{V}$ according to

$$\begin{pmatrix} \widetilde{C}_1^+ \\ \widetilde{C}_2^+ \end{pmatrix} = \mathbf{V} \begin{pmatrix} \widetilde{W}^+ \\ \widetilde{H}_u^+ \end{pmatrix}; \qquad \begin{pmatrix} \widetilde{C}_1^- \\ \widetilde{C}_2^- \end{pmatrix} = \mathbf{U} \begin{pmatrix} \widetilde{W}^- \\ \widetilde{H}_d^- \end{pmatrix}.$$

Note that the mixing matrix for the positively charged left-handed fermions is different from that for the negatively charged left-handed fermions.

The chargino mixing matrices are chosen so that

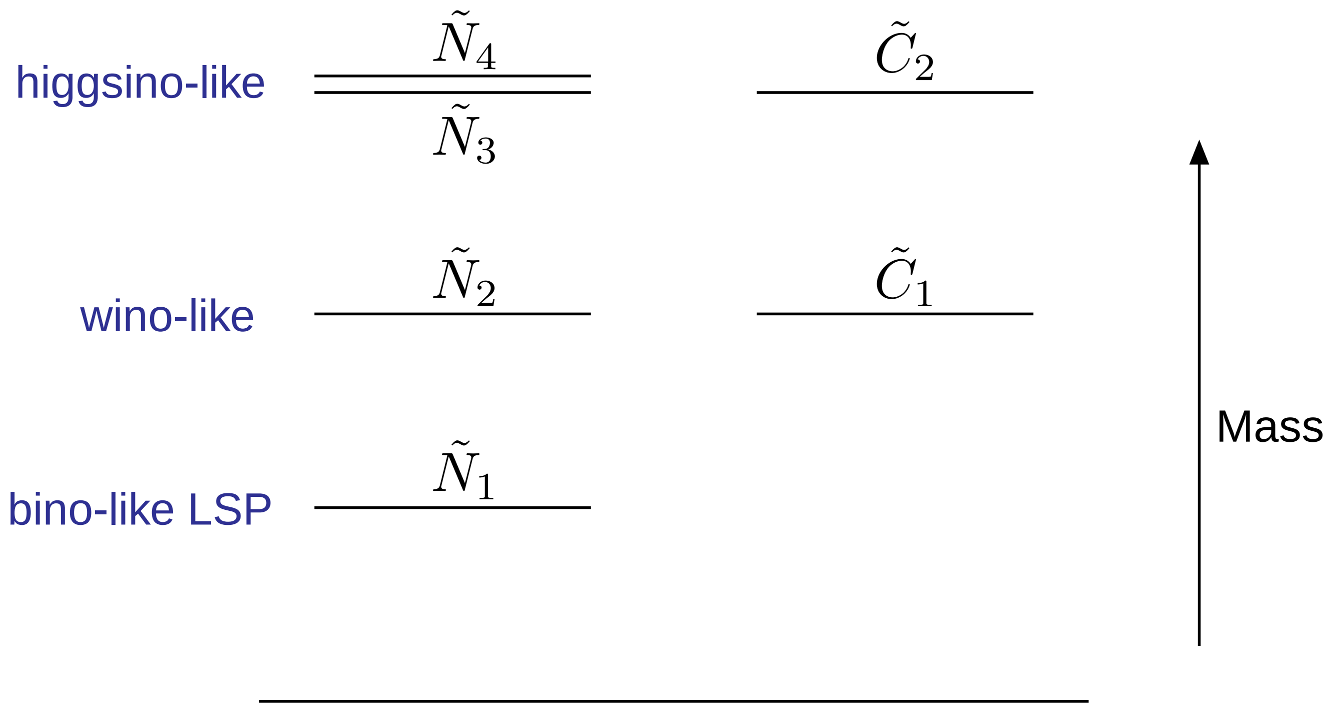
$$\mathbf{U}^*\mathbf{X}\mathbf{V}^{-1} = \begin{pmatrix} m_{\widetilde{C}_1} & 0 \\ 0 & m_{\widetilde{C}_2} \end{pmatrix},$$

with positive real entries $m_{\widetilde{C}_i}$. In this case, one can solve for the tree-level mass^2 eigenvalues in simple closed form:

$$\begin{aligned} m^2_{\widetilde{C}_1}, m^2_{\widetilde{C}_2} \quad = \quad & \frac{1}{2}\Big[|M_2|^2 + |\mu|^2 + 2m_W^2 \\ & \mp\sqrt{(|M_2|^2 + |\mu|^2 + 2m_W^2)^2 - 4|\mu M_2 - m_W^2 \sin 2\beta|^2}\Big]. \end{aligned}$$

In many models of SUSY breaking, one finds that $M_2 \ll |\mu|$, so the lighter chargino is mostly wino with mass close to M_2, and the heavier is mostly higgsino with mass close to $|\mu|$.

A typical mass hierarchy for the neutralinos and charginos, assuming $m_Z \ll |\mu|$ and $M_1 \approx 0.5 M_2 < |\mu|$.



Although this is a very popular scenario, it is NOT guaranteed.

The Gluino

The gluino is an $SU(3)_C$ color octet fermion, so it does not have the right quantum numbers to mix with any other state. Therefore, at tree-level, its mass is the same as the corresponding parameter in the soft SUSY-breaking Lagrangian:

$$M_{\tilde{g}} = M_3$$

However, the quantum corrections to this are quite large (again, because this is a color octet!). If one calculates the one-loop pole mass of the gluino, one finds:

$$M_{\tilde{g}} = M_3(Q)\Big(1 + \frac{\alpha_s}{4\pi}\big[15 + 6\ln(Q/M_3) + \sum A_{\tilde{q}}\big]\Big)$$

where Q is the renormalization scale, the sum is over all 12 squark multiplets, and

$$A_{\tilde{q}} = \int_0^1 dx\, x \ln\big[x m_{\tilde{q}}^2/M_3^2 + (1-x)m_q^2/M_3^2 - x(1-x) - i\epsilon\big].$$

This correction can be of order 5% to 25%, depending on the squark masses! It tends to **increase** the gluino mass, compared to the tree-level prediction.

Squarks and Sleptons

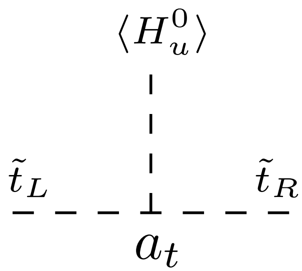
To treat these in complete generality, we would have to take into account arbitrary mixing. So the mass eigenstates would be obtained by diagonalizing:

- a 6×6 (mass)^2 matrix for up-type squarks ($\widetilde{u}_L$, $\widetilde{c}_L$, $\widetilde{t}_L$, $\widetilde{u}_R$, $\widetilde{c}_R$, $\widetilde{t}_R$),
- a 6×6 (mass)^2 matrix for down-type squarks ($\widetilde{d}_L$, $\widetilde{s}_L$, $\widetilde{b}_L$, $\widetilde{d}_R$, $\widetilde{s}_R$, $\widetilde{b}_R$),
- a 6×6 (mass)^2 matrix for charged sleptons ($\widetilde{e}_L$, $\widetilde{\mu}_L$, $\widetilde{\tau}_L$, $\widetilde{e}_R$, $\widetilde{\mu}_R$, $\widetilde{\tau}_R$),
- a 3×3 matrix for sneutrinos ($\widetilde{\nu}_e$, $\widetilde{\nu}_\mu$, $\widetilde{\nu}_\tau$)

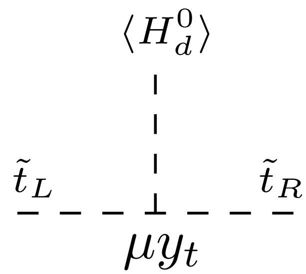
Fortunately, in viable models, most of these mixing angles are very small.

The first- and second-family squarks and sleptons have negligible Yukawa couplings, so they end up in 7 very nearly degenerate, unmixed pairs $(\widetilde{e}_R, \widetilde{\mu}_R)$, $(\widetilde{\nu}_e, \widetilde{\nu}_\mu)$, $(\widetilde{e}_L, \widetilde{\mu}_L)$, $(\widetilde{u}_R, \widetilde{c}_R)$, $(\widetilde{d}_R, \widetilde{s}_R)$, $(\widetilde{u}_L, \widetilde{c}_L)$, $(\widetilde{d}_L, \widetilde{s}_L)$.

For the third-family squarks and sleptons, there are additional effects proportional the large Yukawa (y_t, y_b, y_τ) and soft (a_t, a_b, a_τ) couplings. For the top quark, we have corrections with the diagrammatic representations:



and



The first diagram comes directly from the soft SUSY-breaking Lagrangian, and the others from the F-term contribution to the scalar potential. So, in the $(\tilde{t}_L, \tilde{t}_R)$ basis, the top squark mass^2 matrix is:

$$\begin{pmatrix} m^2_{\tilde{Q}_3} + m_t^2 + \Delta_{\tilde{t}_L} & a_t^* v_u - \mu y_t v_d \\ a_t v_u - \mu^* y_t v_d & m^2_{\bar{\tilde{u}}_3} + m_t^2 + \Delta_{\tilde{t}_R} \end{pmatrix}$$

Therefore, the top-squark system has a significant mixing, with the off-diagonal entries "repelling" the two mass^2 eigenvalues.

Diagonalizing the top squark mass^2 matrix, one finds mass eigenstates:

$$\begin{pmatrix} \tilde{t}_1 \\ \tilde{t}_2 \end{pmatrix} = \begin{pmatrix} c_{\tilde{t}} & -s^*_{\tilde{t}} \\ s_{\tilde{t}} & c_{\tilde{t}} \end{pmatrix} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix}$$

where $m^2_{\tilde{t}_1} < m^2_{\tilde{t}_2}$ by convention, and $|c_{\tilde{t}}|^2 + |s_{\tilde{t}}|^2 = 1$.

In a completely analogous way, there is a non-trivial mixing for the bottom squark and tau slepton states:

$$\begin{pmatrix} \tilde{b}_1 \\ \tilde{b}_2 \end{pmatrix} = \begin{pmatrix} c_{\tilde{b}} & -s^*_{\tilde{b}} \\ s_{\tilde{b}} & c_{\tilde{b}} \end{pmatrix} \begin{pmatrix} \tilde{b}_L \\ \tilde{b}_R \end{pmatrix};$$

$$\begin{pmatrix} \tilde{\tau}_1 \\ \tilde{\tau}_2 \end{pmatrix} = \begin{pmatrix} c_{\tilde{\tau}} & -s^*_{\tilde{\tau}} \\ s_{\tilde{\tau}} & c_{\tilde{\tau}} \end{pmatrix} \begin{pmatrix} \tilde{\tau}_L \\ \tilde{\tau}_R \end{pmatrix}$$

The same sort of mixing occurs for the first- and second-family squarks and sleptons, but is considered negligible because the Yukawa couplings are small, and in most viable models the relevant a-terms are also.

The undiscovered particles in the MSSM:

Names	Spin	P_R	Mass Eigenstates	Gauge Eigenstates
Higgs bosons	0	$+1$	$h^0\ H^0\ A^0\ H^\pm$	$H_u^0\ H_d^0\ H_u^+\ H_d^-$
squarks	0	-1	$\widetilde{u}_L\ \widetilde{u}_R\ \widetilde{d}_L\ \widetilde{d}_R$ $\widetilde{s}_L\ \widetilde{s}_R\ \widetilde{c}_L\ \widetilde{c}_R$ $\widetilde{t}_1\ \widetilde{t}_2\ \widetilde{b}_1\ \widetilde{b}_2$	" " " " $\widetilde{t}_L\ \widetilde{t}_R\ \widetilde{b}_L\ \widetilde{b}_R$
sleptons	0	-1	$\widetilde{e}_L\ \widetilde{e}_R\ \widetilde{\nu}_e$ $\widetilde{\mu}_L\ \widetilde{\mu}_R\ \widetilde{\nu}_\mu$ $\widetilde{\tau}_1\ \widetilde{\tau}_2\ \widetilde{\nu}_\tau$	" " " " $\widetilde{\tau}_L\ \widetilde{\tau}_R\ \widetilde{\nu}_\tau$
neutralinos	$1/2$	-1	$\widetilde{N}_1\ \widetilde{N}_2\ \widetilde{N}_3\ \widetilde{N}_4$	$\widetilde{B}^0\ \widetilde{W}^0\ \widetilde{H}_u^0\ \widetilde{H}_d^0$
charginos	$1/2$	-1	$\widetilde{C}_1^\pm\ \widetilde{C}_2^\pm$	$\widetilde{W}^\pm\ \widetilde{H}_u^+\ \widetilde{H}_d^-$
gluino	$1/2$	-1	$\widetilde{g}$	" "

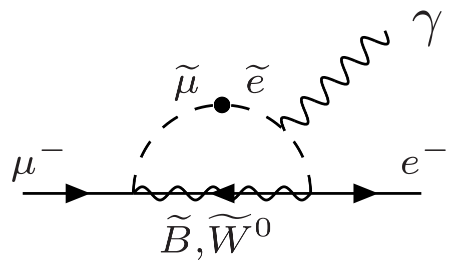
There are 105 new parameters associated with SUSY breaking in the MSSM.

How are we supposed to make any meaningful predictions in the face of this uncertainty?

Fortunately, we already know that the MSSM soft terms cannot be arbitrary, because of experimental constraints on flavor violation.

Hints of an Organizing Principle

For example, if there is a smuon-selectron mixing (mass)^2 term $\mathcal{L} = -m^2_{\tilde{\mu}^*_L \tilde{e}_L} \tilde{\mu}^*_L \tilde{e}_L$, and $\tilde{M} =$ Max$[m_{\tilde{e}_L}, m_{\tilde{e}_R}, M_2]$, then by calculating this one-loop diagram, one finds the decay width:



$\mu^- \rightarrow e^- \gamma$

$$\Gamma(\mu^- \rightarrow e^- \gamma) = 5 \times 10^{-21} \text{ MeV} \left(\frac{m^2_{\tilde{\mu}^*_L \tilde{e}_L}}{\tilde{M}^2} \right)^2 \left(\frac{100 \text{ GeV}}{\tilde{M}} \right)^4$$

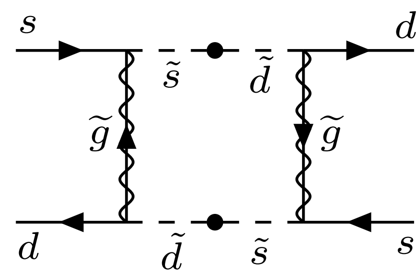
For comparison, the experimental limit is (from MEGA at LAMPF):

$$\Gamma(\mu^- \rightarrow e^- \gamma) < 3.6 \times 10^{-27} \text{ MeV.}$$

So the amount of smuon-selectron mixing in the soft Lagrangian is limited by:

$$\left(\frac{m^2_{\tilde{\mu}^*_L \tilde{e}_L}}{\tilde{M}^2} \right) < 10^{-3} \left(\frac{\tilde{M}}{100 \text{ GeV}} \right)^2$$

Another example: $K^0 \leftrightarrow \overline{K^0}$ mixing:



$K^0 \leftrightarrow \overline{K^0}$

This constrains the flavor-violating SUSY breaking terms:

$$\mathcal{L} = -m^2_{\tilde{d}_L^* \tilde{s}_L} \tilde{d}_L^* \tilde{s}_L - m^2_{\tilde{d}_R \tilde{s}_R^*} \tilde{d}_R \tilde{s}_R^*.$$

Comparing this diagram with the observed Δm_{K_0} gives:

$$\frac{\mathrm{Re}[m^2_{\tilde{d}_L^* \tilde{s}_L} m^2_{\tilde{d}_R \tilde{s}_R^*}]^{1/2}}{\tilde{M}^2} \lesssim 0.001 \left(\frac{\tilde{M}}{\text{500 GeV}} \right)$$

where $\tilde{M}$ is the dominant squark or gluino mass.

The experimental values of ϵ and ϵ'/ϵ in the effective Hamiltonian for the $K^0, \overline{K^0}$ system also give strong constraints on the amount of $\tilde{d}_L, \tilde{s}_L$ and $\tilde{d}_R, \tilde{s}_R$ mixing and CP violation in the soft terms.

Similarly:

The $D^0, \overline{D^0}$ system constrains $\tilde{u}_L, \tilde{c}_L$ and $\tilde{u}_R, \tilde{c}_R$ soft SUSY-breaking mixing.

The $B_d^0, \overline{B_d^0}$ system constrains $\tilde{d}_L, \tilde{b}_L$ and $\tilde{d}_R, \tilde{b}_R$ soft SUSY-breaking mixing.

The soft-SUSY breaking masses must be either VERY heavy, or nearly flavor-blind, to avoid flavor-changing violating experimental limits.

The Flavor-Preserving Minimal Supersymmetric Standard Model

Take an idealized limit in which in which the squark and slepton (mass)^2 matrices are flavor-blind, each proportional to the 3×3 identity matrix in family space:

$$\mathbf{m^2_{\tilde{Q}}} = m^2_{\tilde{Q}} \mathbf{1}; \quad \mathbf{m^2_{\tilde{\bar{u}}}} = m^2_{\tilde{\bar{u}}} \mathbf{1}; \quad \mathbf{m^2_{\tilde{\bar{d}}}} = m^2_{\tilde{\bar{d}}} \mathbf{1}; \quad \mathbf{m^2_{\tilde{L}}} = m^2_{\tilde{L}} \mathbf{1}; \quad \mathbf{m^2_{\tilde{\bar{e}}}} = m^2_{\tilde{\bar{e}}} \mathbf{1}.$$

Then all squark and slepton mixing angles are rendered trivial, because squarks and sleptons with the same electroweak quantum numbers will be degenerate in mass and can be rotated into each other at will. Also assume:

$$\mathbf{a_u} = A_{u0}\, \mathbf{y_u}; \qquad \mathbf{a_d} = A_{d0}\, \mathbf{y_d}; \qquad \mathbf{a_e} = A_{e0}\, \mathbf{y_e},$$

and no new CP-violating phases:

$$M_1,\ M_2,\ M_3,\ A_{u0},\ A_{d0},\ A_{e0} = \text{real}$$

The Higgs mass parameters $m^2_{H_u}$ and $m^2_{H_d}$ are real, and μ and b can be chosen real by convention.

The Flavor-Preserving Minimal Supersymmetric Standard Model (continued)

The new parameters, besides those already found in the Standard Model, are:

- M_1, M_2, M_3 (3 real gaugino masses)
- $m^2_{\tilde{Q}}, m^2_{\tilde{\bar{u}}}, m^2_{\tilde{\bar{d}}}, m^2_{\tilde{L}}, m^2_{\tilde{\bar{e}}}$ (5 squark and slepton mass^2 parameters)
- A_{u0}, A_{d0}, A_{e0} (3 real scalar^3 couplings)
- $m^2_{H_u}, m^2_{H_d}, b, \mu$ (4 real parameters)

So there are 15 real parameters in this model.

The parameters μ and $b \equiv B\mu$ are often traded for the known Higgs VEV $v = 175$ GeV, $\tan\beta$, and sign(μ).

Most viable SUSY breaking models are special cases of this.

However, these are Lagrangian parameters that run with the renormalization scale, Q. Therefore, one must also choose an “input scale” Q_0 where the flavor-independence holds.

What is the input scale Q_0 ?

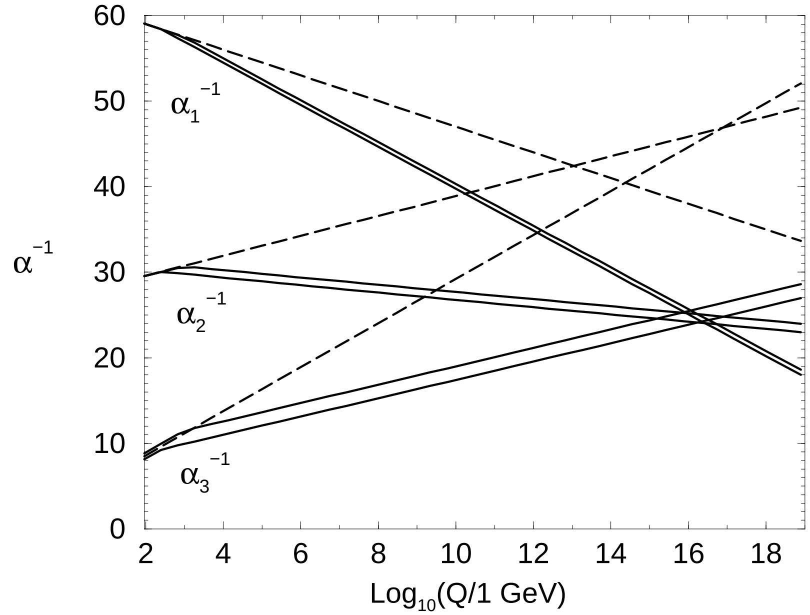
Perhaps:

- $Q_0 = M_{\text{Planck}}$, or
- $Q_0 = M_{\text{string}}$, or
- $Q_0 = M_{\text{GUT}}$, or
- Q_0 is some other scale associated with the type of SUSY breaking.

In any case, one can pick the SUSY-breaking parameters at Q_0 as boundary conditions, then run them down to the weak scale using their renormalization group (RG) equations. Flavor violation will remain small, because the Yukawa couplings of the first two families are small.

At the weak scale, use the renormalized parameters to predict physical masses, decay rates, cross-sections, dark matter relic density, etc.

A reason to be optimistic that this program can succeed: the SUSY unification of gauge couplings. The measured α_1, α_2, α_3 are run up to high scales using the RG equations of the Standard Model (dashed lines) and the MSSM (solid lines).



At one-loop order, the RG equations are:

$$\frac{d}{d(\ln Q)}\alpha_a^{-1} = -\frac{b_a}{2\pi} \qquad\qquad (a = 1, 2, 3)$$

with $b_a^{\rm SM}{=}(41/10{,}{-}19/6{,}{-}7)$ in the Standard Model, and $b_a^{\rm MSSM}{=}(33/5{,}1{,}{-}3)$ in the MSSM because of the extra particles in the loops. The results for the MSSM are in agreement with unification at $M_{\rm GUT} \approx 2\times 10^{16}$ GeV.

If this hint is real, we can reasonably hope that a similar extrapolation for the soft SUSY-breaking parameters can also work.

Origins of SUSY breaking

Up to now, we have simply put SUSY breaking into the MSSM explicitly.

To gain deeper understanding, let us consider how SUSY could be spontaneously broken. This means that the Lagrangian is invariant under SUSY transformations, but the ground state is not:

$$Q_\alpha|0\rangle \neq 0, \qquad\qquad Q^\dagger_{\dot\alpha}|0\rangle \neq 0.$$

The SUSY algebra tells us that the Hamiltonian is related to the SUSY charges by:

$$H = P^0 = \tfrac{1}{4}(Q_1 Q_1^\dagger + Q_1^\dagger Q_1 + Q_2 Q_2^\dagger + Q_2^\dagger Q_2).$$

Therefore, if SUSY is unbroken in the ground state, then $H|0\rangle = 0$, so the ground state energy is 0. Conversely, if SUSY is spontaneously broken, then the ground state must have positive energy, since

$$\langle 0|H|0\rangle = \tfrac{1}{4}\Big(\|Q_1^\dagger|0\rangle\|^2 + \|Q_1|0\rangle\|^2 + \|Q_2^\dagger|0\rangle\|^2 + \|Q_2|0\rangle\|^2\Big) > 0$$

To achieve spontaneous SUSY breaking, we need a theory in which the prospective ground state $|0\rangle$ has positive energy.

In SUSY, the potential energy can be written, using the equations of motion, as:

$$V = \sum_i |F_i|^2 + \tfrac{1}{2} \sum_a D^a D^a,$$

a sum of squares of auxiliary fields. So, for spontaneous SUSY breaking, one must arrange a stable (or quasi-stable) ground state with either $\langle F_i \rangle \neq 0$ or $\langle D^a \rangle \neq 0$, for at least one i or a.

Models of SUSY breaking where

- $\langle F_i \rangle \neq 0$ are called “O’Raifeartaigh models” or “F-term breaking models”
- $\langle D^a \rangle \neq 0$ are called “Fayet-Iliopoulis models” or “D-term breaking models”

F-term breaking is used in (almost) all known realistic models.

This can only happen if the chiral supermultiplet is a singlet.

Spontaneous Breaking of SUSY requires us to extend the MSSM

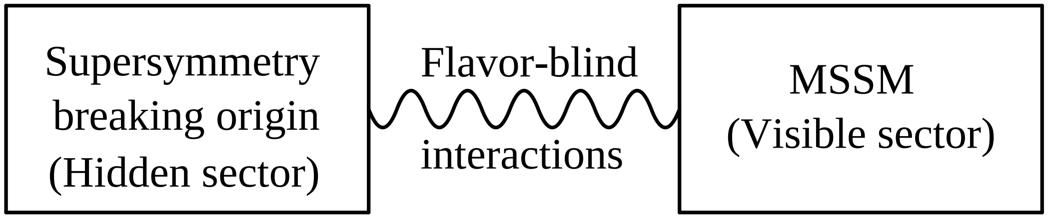
There is no gauge-singlet chiral supermultiplet in the MSSM that could get a non-zero F-term VEV.

Even if there were such an $\langle F \rangle$, there is another general obstacle. Gaugino masses cannot arise in a renormalizable SUSY theory at tree-level. This is because SUSY does not contain any (gaugino)-(gaugino)-(scalar) coupling that could turn into a gaugino mass term when the scalar gets a VEV.

We also have the clue that SUSY breaking must be essentially flavor-blind in order to not conflict with experiment.

This leads to the following general schematic picture of SUSY breaking...

The MSSM soft SUSY-breaking terms arise indirectly or radiatively, not from tree-level renormalizable couplings directly to the SUSY-breaking sector.



Spontaneous SUSY breaking occurs in a "hidden sector" of particles with no (or tiny) direct couplings to the "visible sector" chiral supermultiplets of the MSSM. However, the two sectors do share some mediating interactions that transmit SUSY-breaking effects indirectly. As a bonus, if the mediating interactions are flavor-blind, then the soft SUSY-breaking terms of the MSSM will be also.

By dimensional analysis,

$$m_{\rm soft} \sim \frac{\langle F \rangle}{M}$$

where M is a mass scale associated with the physics that mediates between the two sectors.

Planck-scale Mediated SUSY Breaking (also known as "gravity mediation")

The idea: SUSY breaking is transmitted from a hidden sector to the MSSM by the new interactions, including gravity, that enter near the Planck mass scale M_P.

If SUSY is broken in the hidden sector by some VEV $\langle F \rangle$, then the MSSM soft terms should be of order:

$$m_{\rm soft} \sim \frac{\langle F \rangle}{M_P}$$

This follows from dimensional analysis, since $m_{\rm soft}$ must vanish in the limit that SUSY breaking is turned off $(\langle F \rangle \rightarrow 0)$ and in the limit that gravity becomes irrelevant $(M_P \rightarrow \infty)$.

Since we know $m_{\rm soft} \sim$ few hundred GeV, and $M_P \sim 2.4 \times 10^{18}$ GeV:

$$\sqrt{\langle F \rangle} \sim 10^{11} \text{ or } 10^{12} \text{ GeV}$$

Planck-scale Mediated SUSY Breaking (continued)

Write down an effective field theory non-renormalizable Lagrangian that couples F to the MSSM scalar fields ϕ_i and gauginos λ^a:

$$\mathcal{L}_{\text{PMSB}} = -\Big(\frac{f^a}{2M_P} F\lambda^a\lambda^a + \text{c.c.}\Big) - \frac{k_i^j}{M_P^2} FF^*\phi_i\phi^{*j}$$
$$-\Big(\frac{\alpha^{ijk}}{6M_P} F\phi_i\phi_j\phi_k + \frac{\beta^{ij}}{2M_P} F\phi_i\phi_j + \text{c.c.}\Big)$$

This is (part of) a fully supersymmetric Lagrangian that arises in supergravity. When we replace F by its VEV $\langle F\rangle$, we get exactly the MSSM soft SUSY-breaking Lagrangian, with:

- Gaugino masses: $M_a = f^a\langle F\rangle/M_P$
- Scalar squared massed: $(m^2)_i^j = k_i^j|\langle F\rangle|^2/M_P^2$ and $b^{ij} = \beta^{ij}\langle F\rangle/M_P$
- Scalar^3 couplings $a^{ijk} = \alpha^{ijk}\langle F\rangle/M_P$

Unfortunately, it is **not** obvious that these are flavor-blind!

A dramatic simplification occurs if one assumes a “minimal” form for the kinetic terms and gauge interactions in the underlying supergravity theory. (Whether this assumption is reasonable or not remains controversial.)

This means $f^a = f$ for all gauge interactions, $k_i^j = k\delta_i^j$ for all scalar fields, and $\alpha^{ijk} = \alpha y^{ijk}$ and $\beta^{ij} = \beta M^{ij}$. Then all of the MSSM soft terms can be written in terms of just four parameters:

- A common gaugino mass: $m_{1/2} = f\frac{\langle F\rangle}{M_P}$
- A common scalar squared mass: $m_0^2 = k\frac{|\langle F\rangle|^2}{M_P^2}$
- A scalar^3 coupling prefactor: $A_0 = \alpha\frac{\langle F\rangle}{M_P}$
- A scalar mass^2 prefactor $B_0 = \beta\frac{\langle F\rangle}{M_P}$

This simplified parameter space is often called “Minimal Supergravity” or “mSUGRA”.

The "mSUGRA" parameter space

In terms of the four parameters $m_{1/2}$, m_0^2, A_0, and B_0:

$$M_3 = M_2 = M_1 = m_{1/2}$$

$$\mathbf{m^2_{\tilde{Q}}} = \mathbf{m^2_{\tilde{\bar{u}}}} = \mathbf{m^2_{\tilde{\bar{d}}}} = \mathbf{m^2_{\tilde{L}}} = \mathbf{m^2_{\tilde{\bar{e}}}} = m_0^2\,\mathbf{1}$$

$$m^2_{H_u} = m^2_{H_d} = m_0^2$$

$$\mathbf{a_u} = A_0 \mathbf{y_u}, \qquad \mathbf{a_d} = A_0 \mathbf{y_d}, \qquad \mathbf{a_e} = A_0 \mathbf{y_e}$$

$$b = B_0 \mu.$$

These values of the soft terms should probably be taken at the renormalization scale $Q_0 = M_P$, and then run down to the weak scale. However, it is traditional to use $Q_0 = M_{\rm GUT}$ instead, because nobody has any idea how to extrapolate above $M_{\rm GUT}$! Part, but not all, of the error incurred in doing so can be reabsorbed into the definitions of $m_{1/2}$, m_0^2, A_0, and B_0.

Some particular models can be even more predictive, in principle:

- Dilaton-dominated: $m_0^2 = m_{3/2}^2, \;\; m_{1/2} = -A_0 = \sqrt{3} m_{3/2}$
- Polonyi: $m_0^2 = m_{3/2}^2, \;\; A_0 = (3 - \sqrt{3}) m_{3/2}$
- "No-scale" or "Gaugino mass dominated": $m_{1/2} \gg m_0, A_0$

However, there is no clear theoretical reason why things should be so simple.

The modern viewpoint is to take $m_{1/2}$, m_0^2, A_0, and B_0 as crude, but convenient, parameterizations of our ignorance of SUSY breaking.

It is usual to trade B_0 for the parameter $\tan\beta = v_u/v_d$.
Also, the minimization of the EW potential allows us to eliminate the magnitude (but not the phase) of μ in favor of m_Z.