Physics 661 **Homework 2** **Due Wednesday, January 28, 2009**

Problem 1. Consider the second excited level of atomic hydrogen (the $n = 3$ level). Make a diagram of the relative energy splittings, like the one done in class for the $n = 2$ level (see the bottom of page 18 of the lecture notes). Include numerical values in eV for the fine structure, hyperfine structure, and Lamb shift splittings. You may use the approximate formula

$$\Delta E_{\rm Lamb} = 6.5\delta_{\ell,0}\frac{\alpha^3}{n^3}\left(\frac{e^2}{2a_0}\right).$$

Problem 2. In this problem, we will consider the hyperfine structure in the presence of an external magnetic field $\vec{B} = B\hat{z}$, for the lowest ($n = 1$) level of atomic hydrogen. The hyperfine Hamiltonian as found in class (after taking $n = 1, \ell = 0$) can be written as:

$$H_{\rm HF} = E_\gamma \vec{S}\cdot\vec{I}/\hbar^2 \tag{1}$$

where $\vec{S}$ and $\vec{I}$ are the spin operators for the electron and proton, respectively, and

$$E_\gamma = \frac{8m_e}{3m_p}g_p\alpha^2\left(\frac{e^2}{2a_0}\right) \approx 5.878\times 10^{-6}\,\text{eV}$$

is the energy of the 21 cm line. The electron magnetic moment interacting with the external field gives rise to

$$H_B = 2\mu_B\vec{B}\cdot\vec{S}/\hbar.$$

where $\mu_B = \frac{|e|\hbar}{2m_e c} = 5.788\times 10^{-9}$ eV/gauss is the Bohr magneton. However, we neglect the effect of the proton's magnetic moment interacting with the external field, since it is much smaller. The total Hamiltonian perturbation can then be written as

$$H' = E_\gamma[S_zI_z + \frac{1}{2}(S_+I_- + S_-I_+)]/\hbar^2 + 2\mu_B BS_z/\hbar$$

(a) It is most convenient to work in the state basis labelled by eigenstates of m_s, m_I. Find the 4×4 matrix that represents the Hamiltonian perturbation H' in this basis, and compute its eigenvalues in terms of E_γ and $\mu_B B$. These are the corrections ΔE to the degenerate $1S_{1/2}$ energy level.

(b) Expand your answers for small B, up to quadratic order. Check agreement with the result found in class for the limiting case $B = 0$.
(c) Expand your answers for large B, up to linear order in E_γ. What are the corresponding energy eigenstates in the limit of very large B?
(d) Make a graph of the numerical energy corrections from H', for $0 < B < 1200$ gauss. Label the energy levels near $B = 0$ with $f = 0$ and $f = 1$, and the energy levels for large B with m_s and m_I. (Here f is the quantum number for the total angular momentum $\vec{F} = \vec{S} + \vec{I}$.)