Physics 661 **Homework 3** **Due Wednesday, February 4, 2009**

Problem 1. Consider a particle bound in a simple harmonic oscillator in one dimension. Initially ($t < 0$), it is in the ground state. At $t = 0$, a perturbation to the Hamiltonian is turned on, of the form:

$$H_1(x,t) = Ax^2 e^{-t/\tau} \qquad (t \geq 0)$$

where A and τ are constant numbers. Using time-dependent perturbation theory, calculate the probability that, after a sufficiently long time ($t \gg \tau$), the system will have made a transition to a given excited state. Consider all final states.

Problem 2. A hydrogen atom in its ground state $[(n, \ell, m) = (1, 0, 0)]$ is placed between the plates of a capacitor. At times $t < 0$, there is no electric field due to the capacitor, but for later times the capacitor produces a field:

$$\vec{E} = E_0 \hat{z} e^{-t/\tau}.$$

Using first-order time-dependent perturbation theory, compute the probability for the atom to be found at $t \gg \tau$ in each of the 1st excited states $[(n, \ell, m) = (2, 0, 0)$ and $(2, 1, 1)$ and $(2, 1, 0)$ and $(2, 1, -1)]$.

Problem 3. In this problem you will derive Rabi's formula (see pages 320 and 321 of Sakurai), both exactly and in the approximation of time-dependent perturbation theory.

Consider a two-level system with $E_1 < E_2$. There is a time-dependent potential that connects the two levels, with matrix elements:

$$V_{11} = V_{22} = 0, \qquad V_{12} = V_{21}^* = \gamma e^{i\omega t}.$$

At time $t = 0$, it is known that only the lower level is populated. That is, $c_1(0) = 1$ and $c_2(0) = 0$.

(a) Consider the *exact* differential equation for the coefficients $c_1(t)$ and $c_2(t)$:

$$\begin{aligned} i\hbar \frac{dc_1}{dt} &= V_{12} e^{-i\omega_{21} t} c_2, \\ i\hbar \frac{dc_2}{dt} &= V_{21} e^{i\omega_{21} t} c_1. \end{aligned}$$

where $\omega_{21} = -\omega_{12} = (E_2 - E_1)/\hbar$. Solve these equations for $c_1(t)$ and $c_2(t)$. Do this by assuming a solution of the form:

$$\begin{aligned} c_1(t) &= e^{i(\omega-\omega_{21})t/2}[\cos(\Omega t) - A_1 \sin(\Omega t)]; \\ c_2(t) &= e^{-i(\omega-\omega_{21})t/2} A_2 \sin(\Omega t), \end{aligned}$$

where $\Omega = \gamma^2/\hbar^2 + (\omega - \omega_{21})^2/4$, and A_1 and A_2 are quantities that you will determine in terms of ω, γ, and ω_{21}. Show that the results claimed in eqs. (5.5.21a) and (5.5.21b) of Sakurai follow.

(b) Do the same problem but using the approximation of time-dependent perturbation theory to the lowest non-vanishing order. Compare the two approaches for small values of γ. What happens when ω is very close to ω_{21} ?