Physics 661 **Homework 4** **Due Wednesday, February 11, 2009**

Problem 1. Obtain an expression for the lifetime of the first excited state of the hydrogen atom, $\tau(2p \to 1s)$, by using the formula for the rate of spontaneous emission of electric dipole radiation discussed in class (page 28 of the notes). The lifetime is the reciprocal of the emission rate. Do this for each of the $2p$ states $(n, \ell, m) = (2, 1, 0)$ and $(2, 1, 1)$ and $(2, 1, -1)$. Check that you get the same answer in each case, and that, numerically, $\tau = 1.6 \times 10^{-9}$ seconds. Find the corresponding width of the $2p$ states, $\Gamma(2p \to 1s)$, numerically in eV. Also find the wavelength of the emitted photon and check that the dipole approximation is valid. [Hint: among the quantities you will need to compute is $|\langle(2, 1, 0)|Z|(1, 0, 0)\rangle|^2 = (32768/N)a_0^2$, where N is a certain 5-digit integer that is relatively prime to 32768.]

Problem 1. Do problem 38, page 356 of Sakurai. Hints: Note that the ground-state wavefunction of the 3-d harmonic oscillator can be written as $\Psi(\vec{r}) = \psi(x)\psi(y)\psi(z)$, where $\psi(x)$ is the ground-state wavefunction of the 1-d harmonic oscillator of the same angular frequency. You may find the following definite integral useful:

$$\int_0^\infty r \sin(ar) e^{-br^2} dr = \frac{a\sqrt{\pi}}{4b^{3/2}} e^{-a^2/4b}.$$