Physics 661 **Homework 6** **Due Wednesday, March 25, 2009**

Problem 1. Use first-order perturbation theory to calculate the energies of the ortho and para first excited states $(1s)(2s)$ of the Helium atom. (Experimentally, $E_{\rm para} = -2.146e^2/a_0$, and $E_{\rm ortho} = -2.175e^2/a_0$.)

Problem 2. Consider a spinless particle bound to a fixed center by a central force potential, with simultaneous eigenstates of the Hamiltonian, L^2, and L_z given by $|n, \ell, m\rangle$.
(a) Relate, as much as possible, the matrix elements

$$-\frac{1}{\sqrt{2}}\langle n', \ell', m'|(X + iY)|n, \ell, m\rangle, \qquad \frac{1}{\sqrt{2}}\langle n', \ell', m'|(X - iY)|n, \ell, m\rangle, \quad \text{and}$$
$$\langle n', \ell', m'|Z|n, \ell, m\rangle,$$

using *only* the Wigner-Eckhart theorem. (You may leave your answer in terms of unevaluated but specific Clebsch-Gordon coefficients.) State the necessary conditions for each of the matrix elements to be non-vanishing.
(b) Now, evaluate the above matrix elements in the special case of $\ell = 1$, $m = 0$, and $\ell' = 2$, using the wave function forms $\psi_{n\ell m} = R_{n\ell}(r)Y_{\ell m}(\theta, \phi)$, and writing your answers in terms of unevaluated but specific integrals over r involving the unknown functions $R_{n\ell}(r)$. Use this and your answer to part (a) to infer the values of the relevant Clebsch-Gordon coefficients.

Problem 3. (a) Write xy, xz, yz, $(x^2 - y^2)$, and $3z^2 - r^2$ in terms of the components of an appropriate irreducible spherical tensor of rank 2, $T^{(2)}_{+2}$, $T^{(2)}_{+1}$, $T^{(2)}_{0}$, $T^{(2)}_{-1}$, $T^{(2)}_{-2}$.
(b) The expectation value

$$Q \equiv e\langle \alpha, j, m = j|(3Z^2 - R^2)|\alpha, j, m = j\rangle$$

is known as the *quadrupole moment.* Evaluate:

$$e\langle \alpha, j, m'|(X^2 - Y^2)|\alpha, j, m = j\rangle$$

(where $m' = -j, -j + 1, \ldots, j - 1, j$) in terms of Q and appropriate Clebsch-Gordon coefficients. State necessary conditions for the matrix element to be non-vanishing.