Physics 661 **Homework 8** **Due Monday April 13, 2009**

Problem 1. Consider an experiment in which slow neutrons of momentum $\hbar k$ are scattered by a diatomic molecule; suppose that the molecule is aligned along the y axis, with one atom at $y = b$ and the other at $y = -b$. The beam of neutrons is directed in the $\hat{z}$ direction. Assume the atoms to be infinitely heavy so that they remain fixed throughout the experiment. The potential due to the atoms as seen by the neutrons can be adequately represented by a delta function, so:

$$V(\vec{r}) = a\delta(x)\delta(y-b)\delta(z) + a\delta(x)\delta(y+b)\delta(z).$$

(a) Calculate the scattering amplitude and the differential cross section, in the first-order Born approximation.
(b) How does the quantum result differ from what one would expect classically?

Problem 2. Consider a Hydrogen atom in its ground state. Suppose an incident beam of electrons with $E = \hbar^2 k^2/2m$ scatters off of the classical potential $V(r)$ produced by the atom, which you should convince yourself (using Gauss' law) can be determined from the equation:

$$\frac{dV}{dr} = \frac{Q_e^2}{r^2}\left[1 - \int_0^r dr' 4\pi r'^2 |\psi_{1,0,0}(r')|^2\right].$$

In this problem, the charge on the electron will be denoted by Q_e, to distinguish it from the base of the natural logarithms, $e = 2.71828\ldots$.
(a) Solve the equation above to obtain

$$V(r) = -\frac{Q_e^2}{r}(1 + Ar/a_0)e^{-Br/a_0},$$

where A and B are constants that you will determine.
(b) Find the differential cross section in the Born approximation. (Ignore the fact that the electrons within the target atom are identical particles to the electrons within the incident beam, and neglect the recoil of the Hydrogen atom.) See p. 445 of Sakurai for one way to write the final answer.
(c) In the low energy limit, what is the differential cross section, and how does it depend on θ?
(d) In the high energy limit, show that the differential cross section becomes more and more like a pure Coulomb differential cross section as the energy of the incident electrons increases. Explain why this happens.

(e) For any given fixed energy, find the total cross section. Does the total cross section diverge, as it does for pure Coulomb scattering?

Problem 3. Consider a beam of particles with energy $E = \hbar^2 k^2/2m$ moving in the $\widehat{z}$ direction, which scatter from the attractive Gaussian potential

$$V(\vec{r}) = -V_0 e^{-(r/a)^2}$$

(a) Using the first-order Born approximation, obtain an expression for the total cross-section as an integral over a single parameter. Does it diverge, as was the case for Coulomb scattering? (You do not need to do the integral.)

(b) Obtain another expression for the total scattering cross section by applying the Optical Theorem to the forward-scattering amplitude in the *second*-order Born approximation. [Note that $f(0)$ is real in the first-order Born approximation, as noted in class.] You do not need to do the integrals involved, but you should write your answer in such a way that it is manifestly real and positive.

[Hint for all parts of all problems: `http://integrals.com` can be your friend.]